CHARACTERISTICS OF THINKING PROCESSES OF ELEMENTARY SCHOOL STUDENTS WITH HIGH-CAPABILITY IN UNDERSTANDING MATHEMATICS PROBLEMS

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Abstract

This paper is aimed at analyzing characteristics of thinking processes of the elementary schools students with high capability in mathematics in understanding mathematics problems. Two fifth year elementary school students, male and female, with good capability were chosen as the subject. Data were collected through in-depth interviews and task analysis based on the task of mathematical solution. Data credibility were made by continuous and consistent observations and perseverance improvement, time triangulation and member check. Data were analyzed using a flow model covering data reduction, data presentation and conclusion drawing. The results showed that there is a similarity in the thinking process of female and male students in understanding problems, namely constructing relation in and among elements of what is known, what is asked, giving meaning to important words and phrases, and the use of symbol. For female students, relation is also constructed using contraction, although there is difference of the relation constructed in and among the elements. The relation the female students constructed is richer than that of male students. Characteristics of their thinking processes are grouped into situation and established models.

Key words: Thinking process, Mathematics problem, Understanding mathematics problems, Capability in mathematics

INTRODUCTION

Mathematical thinking capability and using it to solve problems are important objectives of schools. Problem solving is one of objectives from learning mathematics at schools (Depdiknas, 2006) and is the heart of learning mathematics (Pimta, Tayruakham, Nuangchalerm, 2009) due to the fact that by problem solvings, students obtain ways to think, habits to be perseverance, great curiosity and self-confidence in any situation (NCTM, 2000) and methods of developing good thinking skills (Pimta, Tayruakham, Nuangchalerm, 2009). It means that skills and capability are concentration, solving mathematical problems need to be taught to students from basic level. Through mathematical problem solving, students may apply their knowledge and skills to solve more general problems in their lives.

Problem solving is not a uniform and monotonous activity. Problems are not always the same, depending on contents, forms and their processes. This shows that capability in solving problems depends on many factors. Factors influencing problem solving are internal and external, where the former are related to the problems, while latter deals with problem solving. Concerning with individuals serving as problems solvers, factors influencing students’ capability in solving mathematical problems are concentration, which directly or indirectly give influences, attitudes towards mathematics materials, self-respect, teachers’ attitudes and those without direct influence are motivation to achieve and self-help (Pimta, Tayruakham, Nuangchalerm, 2009).

Polya (1973) and Posamentier, Jaye and Krulik (2007) state that the first step in solving mathematical problems is to understand or read the problem. It means that understanding problems
has an effect on the next steps in solving them. To be able to understand problems correctly, understanding of the reading content is also important.

Some researches related to this matter are among others Pape (2004) that studied attitudes in solving mathematical problems among secondary school students using consistent and inconsistent language. Österholm (2006a) studied theoretically the relationship between reading comprehension and problem solving and Oserholm (2006b) characterizes reading comprehension of mathematical texts related to "group" materials at university level. Both Pape (2004) and Osterhom (2006b) employed quantitative statistical tests in their analysis. Moreover, students’ thinking style is really influenced by the capability students have (Albaili, 1997). Gender and capability differences also influence the ways of thinking to solve mathematics problems (Zhu, 2007), meanwhile male and female students’ capability in mathematics is also different (Jensen, 2008; Beaton et al, 1999).

The students with high capability were chosen in order to become one of references by those with low-average capability and may be a model for teachers in facilitating their students in understanding especially mathematics problems and in solving any problems in general.

THEORETICAL BACKGROUND

Understanding Mathematics Problems

Experts in mathematics education state that in the world of mathematics education, a problem is a question or mathematical problem to answer or to respond. But it is also stated that not all mathematical questions are automatically will become problems. A question will become a problem if the question shows a challenge that cannot be solved by routine procedure known to the solver. A Mathematical problem is problem if it is not routine (Hudoyo, 2005) or not standardized one (McNeil dkk, 2006).

Posamentier and Krulik (1998) and Polya (1973) states that "a problem is a situation that confronts a person, that requires resolution, and for which the path to the solution is not immediately known". then Polya (1973) says that there are two types of problems in mathematics, namely problem to find dan problem to prove.

Problem to find is a type of problem which is necessary to be given to students in order to train them about the process of how a concept or principle is found out. Then, Polya (1973) states that problem to find is more important in elementary mathematics, whereas problem to prove is more important in advanced mathematics. From their structures, problem to find and problem to prove may be grouped into well-structure (Jonassen danTessmer, 1997). This problem is mostly found out at schools and universities. They are usually at the end of chapters that need an application of concepts, rules, and principles that have been learned in limited problem situations. These problems have been well defined, the objectives have been known, they are limited or logical operators, and their answers are convergent. The problem of this paper is a type of the problem to find dealing with arithmetics in elementary schools and should be solved.

Polya (1973) and Posamentier and Krulik (1998) define problem solving as follows "...finding a way out of difficulty, a way around an obstacle, attaining an aim that was not immediately understandable". Moreover, Polya explains that problem solving is a psychological process that does not merely involve applications of theorems or propositions learned. According to Polya (1973) and Posamentir, Jaye and Krulik (2007), problem solving in mathematics consists of four main steps, namely (1) understand the problem/ read the problem; (2) devise a plan/ select a strategy; (3) carry out a plan/ solve the problem; and (4) look back.
Based on the stages of problem solving, the first step in problem solving in mathematics is to understand or read the problem. It means that understanding the problem has effects on the next steps in problem solving. In order to be able to understand the problem correctly, understanding of reading content is very important. Questions that may be used by students to understand the problems are as follows:

What is known/asked? Which one is the data? What is the condition? How is to understand? How is to know what is to be known? How is to know the conditions?

1.1 Thinking Process in Understanding Problems

When we imagine something or try to solve a problem, we call it thinking. Thinking is needed in decision making and problem solving. Thinking is the highest mental activity existing in human beings. Thinking is defined as a process of producing a new mental representation through the transformation of information that involves complex interactions among mental attributes such as evaluation, abstraction, reasoning, imagination, and problem solving (Glass dan Holyoak, 1986; Solso, 1995). It is in line with Mayer’s opinion (in Solso, 1995) that thinking covers three main components, namely (1) thinking is a cognitive activity that happens in one’s mind, which cannot be seen, but which can be concluded based on the visible behaviors, (2) thinking is a process that involves some manipulations of knowledge in the cognitive system. Knowledge stored in memory is combined with present information so that it changes one’s knowledge about the situation he is facing, and (3) thinking activities are led to produce problem solving.

Based on some opinions above, thinking in this present study is all mental activities that may be observed from visible behaviors in the forms of statements and writings in understanding mathematical problems.

Based on the term of thinking, in order to explore individual mind, there are some ways to do, namely using think out loud method or think aloud and task analysis (van Someren et al, 1994; Calder & Sarah, 2002). The think aloud method is done by asking students pronouncing aloud when they are solving a problem and what is pronounced may be repeated if necessary during the process of problem solving. It is intended to make the subjects may tell what they are thinking.

Besides think aloud method, in order to see ideas in one’s mind, task analysis may be employed (van Someren dkk, 1994). The work the student did is a form of visualization or verbalization of knowledge he possesses in responding each information or problem he faces. One’s thinking to understand a problem may be seen from his visible behaviors, either his expressions or writings when he understands the problem.

When understanding a problem, it is of course not merely to read, but also to digest the materials presented and to understand what is happening. Understanding/reading a problem is an activity of identifying what is to be asked to be solved and the facts presented.

Reading as an active process where the reader interacts with the reading passage to construct meaning. Understanding is a matter to activate or to build a scheme as a coherent explanation of objects or events mentioned in the reading passage (Anderson, 1984). This means that reading a passage, the mental representation of the reading material is constructed or built by the reader, depicting how the reader understands the reading passage. Some studies on reading comprehension show or support a conclusion that there are levels of mental representation when read a reading passage (Österholm, 2006b; Van Dijk dan Kintsch, 1983). The levels of the mental representation are: surface component, textbase, and model of situation. The mental representation at the surface
level always exists when one is reading, since it generally happens and this does not give meanings of words and phrases. At this level, at least there are some words and phrases to be remembered, even when one understands the meaning of texts. Textbase represents the meaning or sense of the reading consisting of elements and relations directly obtained from the reading itself, without any addition which does not explicitly exist in the reading. Meanwhile a construction integrating textbase and other relevant aspects of the reader’s knowledge is called a model of situation.

Problem solving has been studied from various perspectives, such as text processing (Kintsch, 1994; Van Dijk dan Kintsch, 1983), information processing (Silver, 1987; Mayer, 1992) and schemata theory (Marshall, 1995). All the perspectives agree that understanding problems is very essential for problem solving. Österholm (2006a) made literature study on the relationship between reading comprehension and problem solving. The results of his study showed that 1) reading processes may influence the process of problem solving, but may also serve as a part of solution processes, 2) the situation of problem solving influences the reading process, depending on the readers’ previous experiences in the same situation. Österholm (2006b) examined characteristics of reading comprehension at university level using mathematical text with and without symbols. Pape (2004) studied behaviors of secondary high schools students in solving mathematical problems by making use of consistent and inconsistent languages. There are little researches of student thinking processes in solving mathematical problems in a perspective of reading comprehension (Pape, 2004). This paper is focused on analyzing the thinking processes of male and female elementary students with high capability in mathematics in understanding mathematical problems.

METHOD

Subject
The subject of this present research is the fifth year elementary school students with high capability in mathematics at the age of 10 to 11 years. High capability in mathematics is seen from the results of scores of mathematical tests. Students are classified as high capability in mathematics if their scores at least 80 from the range of 0 – 100. The test results showed that there are 5 students (16.13%) with high capability in mathematics (16, 13%), 2 male and 3 female students from 31 students. Then two students, male and female, with relatively scores were chosen as the subject of this present research.

Instrument
Instrument in this research consists of the main instrument, namely the researchers themselves and supporting instruments including audiovisual recorder, numeracy test (NT), task of mathematical problem solving (TPS), and interview guide. The NT was constructed by adopting test items of the final examination for elementary students in the form of multiple choice which were then changed into story test in line with the content standard of the 2006 mathematics curriculum for fifth year students, especially the odd semester.

As in the NT, to construct the TPS was preceded by studying the content standard of the 2006 mathematics curriculum for fifth year students and test instruments used by previous researchers in exploring student thinking. The instrument refers to the one developed by Stephens (2008) and Stephens and Wang (2008). Then the researchers asked for permission to Stephens to use the test he has developed, but the test was changed into the story form. Based on his permission and suggestions, the test was furtherly developed either in terms of the data given or the questions. The
arranged answer sheets (the NT and the TPS) were validated by elementary mathematics teachers that had possessed certificate of educators, by experts in mathematics education and experts in evaluation in terms of the content of the test and of the language used. Based on the results of validation, a readability test was informally done to two fifth year students, male and female. The result was that the two students were able to mention what is known and what is asked, which are two important matters in the test of problem to find (Polya, 1973). In this research, two types of equal numeracy test were developed, each is called NT II and NT II.

Then, interview guide was developed in order to help dig out the subject thinking process. This guide refers to understanding problems of the first level of Polya’s mathematical problem solving.

**Data and Data Credibility**

Based on the NT I and NT II the subjects had developed, data, either from the interviews or results of the subjects’ work in understanding problems, were obtained. The mechanism of collecting data, either in the task of problem solving II and II began by asking to subjects to read the task of problem solving which was continued to in-depth interviews, to write what was understood of the problems and it was then continued to in-depth interviews on the basis of the results of the writing. The data were video recorded. To assure the credibility of the obtained data, continual/consistent and perseverant observations (improving perseverance), time triangulation and member check were made (Moleong, 2011; Sugiyono, 2011).

**Data Analysis**

On the basis of the credible data, an analysis using a flow model consisting of three flows of activities that happen simultaneously namely data reduction, data presentation and conclusion drawing, was made (Miles & Huberman, 1992). In this case is the characteristics of thinking process of the elementary school students with high capability in mathematics in understanding mathematical problems.

**RESULTS AND DISCUSSION**

**Results**

Characteristics of female students’ thinking process (FS) in understanding mathematical problems are to build relations in and among five core elements, namely, what is asked, meanings of words or phrases, use of symbols and use of contraction. This is delineated from the quotation of the interviews results as follows:

\[ R \text{ After reading the problem, what do you understand of the problem?} \]
\[ FS \text{ The number of candies in the container (while paying attention to the problem using the pencil)} \]
\[ R \text{ What else?} \]
\[ FS \text{ (paid attention to the problem and pointed to the problem and then said): “the same direction”} \]
\[ R \text{ What direction?} \]
\[ FS \text{ (Reading the problem): The number of candies in the red container is two time more than those in the green container, the number of candies in the blue container is not more than those in the green container.} \]
\[ R \text{ From what you have mentioned, are the number of candies and the} \]
instruction known or asked?

FS  What is known

R  O, yeah. What is meant by “not more than “ in the direction?

FS  SO the number of candies in the blue container is not more than the total number of those in the green container (paying attention to the problem)

R  What is meant by not more than?

FS  (kept silent and paid attention to the problem and said): the number is not more, but may be less

R  What is asked?

FS  (Read the problem): How many means the number of candies with milk taste in the green container? The same number means the number of candies with melon taste yang might be in the blue container?

Picture 4.1: Results of hand writings on FS’s Problem Understanding

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Information:
TSPA11001: contractions used by FS
TSPA11002: symbols used by FS
TSPA11003: pictures as the substitution of the word container
TSPA11004: what is known
TSPA11005: What is asked

Based on the results of the writing, in understanding problems, FS writes using her own language. When writing what is known, he has been able to identify something to be asked by writing “rs susu = ?(milk taste) and rs melon = ?” (melon taste). This means that she has connected dit with the knowledge or schemata she has possessed before.

What is known is obtained from the information in the problem dealing with numbers and instruction in the form of limitations. What is answered is also obtained from information in the problem concerning with what is not known yet and the sentence of asking. The sentence of asking relates to the question words and question mark. The use of symbols and making meanings of words or phrases deal with comprehension of the words or phrases existing in the problem and knowledge possessed before. Not all important words or phrases are made their meanings by FS. Meanwhile, the use of contractions relates to previous knowledge and information in the problem. Then, there is a relation between what is known and what is asked, namely making counting operations to what is known to answer what is asked and between the use of symbols by asking meanings of words and phrases in the problem. This is shown from the quotation of the results of interviews below:

R From where do you know that what you have mentioned is what is known?
FS numbers and instruction
R Ok. Where is the question you have mentioned from?
FS from the question
R What do you mean?
FS From the sentence of the problem
R What is the sentence about?
FS Asking
R Where do you known the sentence „asking“?
FS From the word how many and question mark
R O, yeah. Is there anything else that makes you know what is asked?
FS (kept silent and paid attention to the problem and then said) “what has not been known”.
R What is meant by ‘what is possible’? In the question?
FS (Spoke softly of what is possible, kept silent and paid attention to the problem and said) “possible number”.
R What is meant by possible?
FS Not understand (while smiling and shaking head). The number of candies in the blue container is not more than those in the green container, but those in the blue container may be less than those in the blue one
R Ok, is there any relation between what is known and what is asked?
FS Yes. To answer what is asked needs what is known.
The characteristic of male students’ thinking process in understanding mathematical problems is by constructing a relation in and among four core elements, namely what is known, what is asked, meaning words or phases and using symbols. This can be seen from the quotation of results of interviews as follows:

R After reading the problem, what do you understand from the problem?
MS (kept silent, then read the problem): Each container contains two taste candies with the same form and size. The red container contains 81 strawberry candies and 47 lemon candies. The green container contains 23 coffee candies and the rest, milk candies. The blue container contains 46 pineapple candies the rest melon ones. The number of candies in the red container is two times than that of the green container. The number of candies the blue container is not more than the number of candies in the green container.

R From what you have mentioned, what is known or what is asked?
MS (kept silent and then said): “what is known”.

R What is known?
MS (directly read the problem): The red container contains 81 strawberry candies and 47 lemon candies. The green container contains 23 coffee candies and the rest, milk candies. The blue container contains 46 pineapple candies the rest melon ones. The number of candies in the red container is two times than that of the green container. The number of candies the blue container is not more than the number of candies in the green container.

R What is meant by “not more than”?
MS Not more than the number mentioned

R What is meant by not more?
MS Not more than the number mentioned (after kept silent and looked at the problem)

R Is it allowed if it is the same?
MS Ok

R Then, what is asked?
MS (Read the problem): How many candies with milk taste are in the green container? And how many candies with melon taste might be in the blue container?
Gambar 4.2: Results of Handwriting of the MS’ Problem Understanding

Information:
- TSPA11001: construction used by MS
- TSPA11002: what is known
- TSPA11003: what is asked

What is known is obtained from information in the problem that deals with numbers and direction to do counting operations. His understanding of the problem is identical with what is known, what is asked is also obtained from information in the problem that is related to the question word and dan question mark. While the use of symbols and making meaning of words and phrases deal with understanding of words and phrases in the problem and knowledge previously possessed. Not all important words and phrases are understood by the MS. Then, there is a relation between what is known and what is asked, namely, what is known is used to answer questions and relations between the use of symbols and making meaning words and phrases in the problem. This is shown in the quotation of the results of interviews below:

R  What information in the problem that states what is known?
MS  (Kept silent, and than said): information to add, reduce, multiply or to divide
R  O, yeah. Is there any more information stating what is known?
MS  (Kept silent and looked at the problems and then said): there are numbers
R  Is there anything else to be asked?
R  What is meant by what is possible in the second question?
MS  (Kept silent, put the right hand to support the chin and looked at the problem and said): “not know”
R  Where do you know that it is what is asked?
MS  From the problem
R  What information in the problem signing that what you mentioned is what is asked
MS  There is question mark, interrogative word how many.
R  Is there any relationship between what is known and what is asked?
MS  Yes. What is known is used to answer the question.

Discussion

The FS’ thinking process in understanding mathematical problems builds relation in and among what is known, what is asked and making meanings of words and phrases, use of symbols and of contractions. Meanwhile the MS’s thinking process in understanding mathematical problems builds relations in and among what is known, what is asked, making meaning of words and phrases and use of symbols. It means that the relation constructed by female subject in understanding problems is
richer than that of male subject. It is also the case on the relation constructed in understanding what is asked. It is inline with an opinion that male students perform better than female ones in test items of multiple choice ad female students perform relatively better than male students in essay/story tests (Bolger & Kellaghan, 1990). Any reason that there might be differences in sex is a fact that story/essay test items needs verbal competence, and some multiple test items do not need such a verbal competence (Murphy, 1982).

Then, what is related to the FS and MS in understanding what is known and what is asked is limited to information from the problems. Such an understanding in the perspective of reading comprehension includes the second level (text base), namely reading comprehension which is merely based on what is on the reading texts/passages (Österholm, 2006a; Van Dijk and Kintsch, 1983). But the MS and FS’s thinking process in using symbols have integrated information in the problem (reading texts) and their previous knowledge. Such an understanding in the perspective reading comprehension (text) is included at the third/highest level (model of situation). (Österholm, 2006a; Van Dijk dan Kintsch, 1983). FS and MS have made various relations in understanding problems; they may able to specify relations of what is known, what is asked, making meanings of important words and phrases and the use of symbols, which are important elements in understanding problems (Polya, 1973; Posamentier, Jaye and Krulik, 2007). The thinking process of this kind is categorized into an established group (Stephens and Wang, 2008).

CONCLUSION AND REMARK

The characteristic of female and male students’ thinking process in understanding problems is that they construct a relation in or among core elements of understanding problems, namely what is known, what is asked, making meanings of important words or phrases and the use of symbols. Characteristics of their thinking processes in understanding problems may be grouped into a model of situation, by combining their previous and established understanding of the reading content and knowledge. However, the relation they both built is different in or among the core elements of understanding of problems.

Understanding problems is an early stage in mathematical problem solving. This stage will influence the next stages, success or failure in solving problems. As a result, it is important for mathematics teachers to facilitate their student in understanding problems by among other developing their thinking processes. The discussion of this paper is limited to problem understanding in the stages of mathematical problem solving developed by Polya and limited to elementary school students with high capability in mathematics. Therefore, it is necessary to study the thinking process in other stages such as making a plan, doing the plan or restudying elementary, secondary or university students with high, average and low capability in mathematics.

REFERENCES


