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DESIGN RESEARCH: RATIO TABLE AND MONEY CONTEXT AS MEANS TO SUPPORT THE DEVELOPMENT OF STUDENTS'PROPORTIONAL REASONING

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Abstract

In general, students use the algebra notation and cross multiplication to solve proportional problems. However, they may just memorize the procedure without understanding the mathematical concept behind it. In that case the learning process remains meaningless. Therefore, we designed a learning sequence based on the tenets of Realistic Mathematics Education (also known as PMRI in Indonesia), which can help the students to develop proportional reasoning. Twenty nine students of grade 4 in SDN 179 Palembang were involved. They did activities which used money as the context and the ratio table as the tool to do calculation and reasoning. This study is part of a Master Program research which aims to investigate how the ratio table can support the development of the students' proportional reasoning. But, this paper will focus only on how the ratio table together with the money context can help the students to develop proportional thinking. Design research was used as research methods for this study. The result shows that the money context together with the ratio table may support students to develop their proportional understanding.

***Keyword**: PMRI, Design Research, Ratio Table, Proportional Reasoning, Contextual Problem, Money Context.*

INTRODUCTION

In our daily life we may notice that the concept of proportion is applied in everywhere. If we buy three books for Rp10, 000 then we need Rp30, 000 to buy nine books. If we need 1.5 kg flour to make two pans of cake then we need 3 kg in order to have four pans of cake. It makes the proportion as an important concept for daily life. However, the concept of proportion is difficult (Tourniaire & Pulos, 1985).

Often students learn to solve proportional problems in a formal way. They use the algebra notation and do cross multiplication. This, however, is not meaningful for them; sometimes they just memorize the procedure without understanding the reason behind it. In addition, if cross multiplication is not fully understood by the students this may lead to some incorrect strategies (Lesh, Post and Behr, 1988 as cited in CPRE, CAMS & El Paso). Thus, the learning process should not only emphasize how to solve the problems but it should also develop the students' proportional reasoning.

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Langrall & Swafford (2000, p. 260) state that the instruction on proportional reasoning should start with situations which can be visualized or modeled. One of the models that can be used in learning proportions is the ratio table. It helps the students to make handy calculations, to gain insight, and to reason with proportions because the table engages the students to write intermediate steps as well. For instance, to find how many liters of milk which are needed to make 80 glasses of strawberry milkshake, if we know that we need 15 liters to make 60 glasses, the students may use 5 for 20 as intermediate step or 30 for 120 then 10 for 40 to get 20 for 80. The strength of a ratio table is that the students can reason with a number relationship which they already know (van Galen et al, 2008).

In this study, we will focus on the question how the ratio table and the money context can support pupils to develop proportional reasoning. 29 students of Grade 4 participated in this study. The present study is part of a Master Program Research which aims to investigate how the ratio table can support the development of the students' proportional reasoning. The activities are designed based on the tenets of Realistic Mathematics Education.

Moreover, this study aims to develop instructional activities, a local instruction theories and a domain-specific instruction theory as well. So, design research is used as the research approach in this study.

THEORETICAL BACKGROUND

Proportion

Proportion is defined as the equality of two ratios, for example $\frac{a}{b} = \frac{c}{d}$ (Tourniaire & Pulos, 1985; Langrall & Swafford, 2000). In Karplus et al (1983), Tournaire & Pulos (1985) and Silvestre & da Ponte (2012) proportional problems are categorized in two types: missing value problem and comparison problem. Missing value problems ask for the fourth number by presenting the three numbers, such as how many chocolates that we get for Rp7, 500 if we know that 4 chocolates cost Rp3, 000. Comparison problems present two or more pairs of numbers and ask about their comparison. For instance, if there are two kinds of sodas and soda A costs Rp4, 000 for 3 cans while soda B costs Rp3, 000 for 2 cans, we can say that soda A is cheaper than soda B.

Proportional Reasoning

Proportional reasoning is defined as a term that shows reasoning in a system of two variables between which there exists a linear functional relationship (Karplus et al, 1983). Tournaire & Pulos (1985) states the factors that may influence students' performance as follows.

Structural variables

- The number structure of the proportion problems influences the subjects' performance. There are three main difficulty factors: presence of an integer ratio, numerical complexity, and the order of number that one is looking for. The presence of integer ratios makes a problem easier. The difficulty introduced by non-integer ratios is widely recognized.
- The presence of a unit.
- The presence of unequal ratios, for comparison problem.

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Context variables

- The familiarity of the context. A familiar problem is easier than unfamiliar ones.
- Mixed problems are more difficult than non-mixed problems.
- People can more easily visualize discrete content than continuous content.
- The mode of delivery of the problem.

Context

One of the tenets in Realistic Mathematics Education is the use of real context (Gravemeijer, 1994). While the students apply mathematical concepts to other aspects on their daily life, they reinforce and strengthen the concept (Zulkardi, 2002). Furthermore, Tournaire & Pulos (1985) stated that the proportional problem with a familiar context is easier than unfamiliar ones. Therefore, in this study we used the money context which is real and familiar for the students.

Ratio Table

The other tenet of Realistic Mathematics Education is that in the learning process models are important as a bridge from informal into formal mathematic (Gravemeijer, 1994). In this study, we will use the ratio table as the model to develop the students' proportional reasoning. The ratio table is introduced as a systematic list that students can use to do the calculations for solving proportional problems. In the end, the students will use the ratio table as a tool to think at a more formal level.

Why did we choose the ratio table as a model? Because it helps the students to make handy calculations, to gain insight, and to reason with proportions because the table may let the students to write down intermediate steps. The ratio table shows the proportion clearly. Using the ratio table, we can expand the number of rows/columns to suit our needs. The strength of ratio table is that the students can reason with number relationship which they already know (van Galen et al, 2008).

RESEARCH METHOD

Twenty nine students in grade 4 (9-10 years old) participated in this study. At the start of the study, the researcher conducted a pretest and interviewed the participants. All interviews were video-taped. In the whole study we implemented five activities which were designed based on the tenets of Realistic Mathematics Education. In this paper we will focus on two activities which used money as the context, the second and the fifth activity, in order to observe how the context of money together with the ratio table can build up the students' proportional reasoning. Design Research was used as the research method of this study in order to go in depth to the students' thinking as well as to contribute to the development of the instructional activities, the local instruction theory and the domain-specific instruction theory. All the videos of the learning activities in the class, field notes, and the students' works were collected as the data collection. We constructed a hypothetical learning trajectory (see, for reference, Simon, 1995, or Simon & Tzur, 2004) as a guideline for the analysis of the data.

RESULTS

Exploring the Ratio Table

An important session in this learning sequence let students explore the ratio table. The students need time to reinvent what kind of strategies that can be used. These strategies will be useful for the students to do calculations with the ratio table.

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In this activity, the students were asked to make a price list of chocolates if it is known that 4 chocolates cost Rp3, 000. The students had the freedom to choose the number of chocolates themselves, and then they had to find the price of these chocolates.

During the discussion in group, the researcher found out one of the interesting discussion. It shows that the ratio table may lead the students to gain different strategies. The money context leads them to reason why they can use that strategy as well.

In the following fragment we see how one of the students- Belda- was using an addition strategy, whereas the other one-Raihan-used a strategy of doubling.

From the previous activity, the students learned about repeated addition. Thus, many students, included Belda, used repeated addition to determine the number of chocolates and found its price. For instance, for 8 and 12 chocolates she did 4+4 chocolates costs Rp3, 000+Rp3, 000 and 8+4 chocolates costs Rp6, 000+Rp3, 000. However, she was confused about how she could find the price of 24 chocolates. She tried to calculate by using her fingers but still couldn't find the answer. Raihan also tried to figure out and by seeing the table which showed the list of the price, he got the idea to double the price (line 8, 16-18, 20, 21). He could think that the price of 12 chocolates was Rp9, 000, so the price of 12 more chocolates was Rp9, 000 more. It means that he realized about the proportion of every 12 chocolate costs Rp9, 000 and he developed it into doubling strategy.

Belda : (The price of) 12 chocolates is 9, 000
Raihan : 18 (thousands)
18, 000
Belda : Add (three thousands) more (is) twelve thousands.
Raihan : 18, 000
Belda : Add more… so… [*counting with fingers and mumbling*]
Raihan : 18, 000
Look… It is 12… add all. 12 more.
18 (thousands)
Belda writes the 18 ,000 as the price of 24 chocolates
Raihan : 48 thousands… 48 chocolates [*Raihan corrects the name of item*]
48…that 48 plus 18…36 [*It should be 18 plus 18*]
Belda : No. How was your way? (…?) [it is not clear what Belda said]
Raihan : 18… Here…24 plus 24 [*Raihan pointing the 18, 000*]
Belda : Oh… So?
Raihan : 18 (thousands) plus 18 (thousands) is 36 (thousands)
24 (chocolates) plus 24 (chocolates)
36 (thousands)
Belda : How?
Raihan : It says that 24 (chocolates) is 18 (thousands).
So, 18(thousands) plus 18 (thousands).
Belda : 18 (thousands) plus 18(thousands)…
Raihan : 36(thousands)

The context helps the students to think about proportional reasoning (if you buy 12 more, you should pay Rp9, 000 more). In addition, the ratio table helps the students

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to organize the information so that they may realize about the proportion. So, the ratio table together with the context helps the pupils to develop the proportional reasoning.

The money context may remind the students to think proportionally

It is necessary to expand the context for the proportional problem, so that the students don't limit their idea of proportionality just in money context. In the following activity, the students were given a problem as follows.

In the second hand shop, we can change 5 old magazines with 4 story books. How many story books we get for 35 old magazines?

The following fragment shows how the money context may remind a student of the rule of proportionality when the students works for another context.

Nasywa : It means that if 6 (old magazines) you get 8 (story books) right?
Researcher : Why? 5 (old magazines) get 4 (story books)…If 6 get 8? Where did it come from?
Nasywa : I added it.
Researcher : Added by 1?
Nasywa : Yes, it added by 1 and this… 4 added by 4 [*Pointing the number of story books*]
Researcher : This is added by 4 [*pointing the number of story books*], but why this is added by 1 [*pointing the number of old magazines*]?
Nasywa looks confused and cannot give the answer
Researcher : It said that for 5 old magazines you get 4 story books…
Nasywa : So, it also added by 4? [*pointing the number of old magazine*]
Researcher : Really? See, first you have 5 (old magazines) and it can be changed with 4 (story books).So, if you just add 1 (magazine)…do you think the shop will add 4 (story books)? If so then the shop will be in loss.
Nasywa : So, it means that it added by 5? [*not clear which one is added by 5 but it seems that she talked about the story books*]
Researcher : What do you think? The shop wants to be loss? It is similar with when you were buying, like the money yesterday. You bought for Rp 3000 and got 4 chocolates and you wanted to buy 5 chocolates and should pay Rp 6000. Do you want it?
Nasywa : Buy 5 chocolates…? No, I don't.
Researcher : Why? You are loss, aren't you? You just add 1 more chocolate but you should pay Rp3000 more… Whereas, if you pay Rp3000 more, how many chocolates you should get? 4 more chocolates right? Now, you have 5 magazines and you get 4 story books. Then, if you want to get 4 more story books. How many magazines you should add?
Nasywa : Five

As Nasywa learned from the previous lesson, she used the repeated addition to solve this problem. However, she didn't do it in the right way. At the first, Nasywa added 4 more story books every time she added 1 magazine. But after the researcher reminded her of the money context (line 18-21; 23-25), she understood that the

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repeated addition should be done as a ratio. Therefore, we can say that the money context may be used to remind the students of the rule in proportionality.

Best Buying

The last activity of the learning sequence aimed to teach the students the idea of proportionality in comparison problems. A suitable context that can be used in this activity is choosing the best price because it is common for the students to look for the cheaper price when they buy something which is of the same quality but has a different price and quantity.

The teacher asked the students to work together in group of two or three and solve the following problem.

Best Price

1. From the following sodas, which is the cheaper one? Explain your reason!

Figure 1. Comparison problem

The group of Livia and Hudi is one of the groups who used the ratio table to help them made a decision which soda is the cheaper one.

Soda A	Harga	Soda B	Harga
3	RP 4.000	2	RP 3.000
6	RP.8.000	4	RP 6.000
9	RP.12.000	6	RP 9.000
12	RP.16.000	8	RP.12.000
15	RP.20.000	10	RP15.000
18	RP 24.000	12	RP.18.000
21	RP 28.000	14	RP.21.000
24	RP 32.000	16	RP.24.000
27	RP 36.000	18	RP.27000
30	RP40.000	20	RP.30.000

Figure2. The table for soda A and soda B from the group of Livia and Hudi

The following conversation was part of their discussion.

Livia : How much for each? [*pointing the price of soda A, Rp4000 for3 sodas*]. It should be divided…
Hudi : It is not divisible [*Take out a book to write the calculation*]
Livia : Soda A. We should divide first…
Hudi : [*Do the division of 4000 by 3*] It remains Liv… What I said. It remains 1.
Livia : So, it starts with 3 right? [*Make the ratio table*]

About 5 minutes later

Livia : So, which one is cheap?
Hudi : We should look for the equality. Twelve and twelve. This is cheap [*pointing soda A*].
Livia : 16 (thousands) isn't it?
Hudi : Yes, and this is 18 (thousands) [*pointing soda B*].
Livia : Yes, that's right. For 18, this is 24 (thousands)…and this 18… Which one is cheaper?
Hudi : I don't know… This is cheaper. [*pointing soda A*]
Livia : It is until 30 [*she is pointing the table for soda A*]. [*Pointing the soda B*] For 30 (sodas B), how much? [*She asks Hudi to continue until 30*] add 2, add 2, add 2… Use that book first. [*Instead of writing directly on the worksheet, they chose to write on the book first because the teacher asked them to write in the worksheet neatly*]
Researcher : [*Hudi takes out his book to do the calculation*] Just make it behind.
Hudi : So, we make a table again then?
Livia : Yes. We make until…
Hudi : Until 100 [*kidding*]
Livia : 30.
Researcher : Is it necessary to continue the table?

Livia and Hudi are just smilling

Livia : [*she whispers to Hudi*] Is it necessary?
Researcher : From the table here, can you say which the cheaper one is?
Hudi : This [*pointing soda A*]
Livia : This [*she agrees with Hudi*]
Researcher : If so, why you want to continue the table?
Hudi : [*Livia smiles then she writes the answer*]…

The above fragment describes a pair of students who try to solve the problem of comparing the price of two sodas. Known that 3 sodas A cost Rp4, 000 and 2 sodas B cost Rp3, 000. Initially, Livia wants to solve the problem by looking the price of 1 soda for each type. But Hudi says that they cannot divide the 4, 000 by 3 because there will be a remainder of 1. Maybe he also thinks that 1, 333 is not a regular price. Thus, Hudi suggests making a table of the price for each soda as they learned on the previous lesson. After they have finished making the table - they make a long table, it's more than what they need (see figure 2)- they discuss how to determine which is the expensive one and the cheaper one. Hudi gets an idea that they can find out which the cheaper soda is by comparing the price of the equal quantity (line 9). He sees both lists and realizes that soda A is cheaper than soda B (line 9,10).

However, the way they used the table to figure out the cheaper sodas was not efficient. As mentioned before, the ratio table which they wrote was already more than they need (see figure 2) to determine the cheaper soda but Livia was still looking for the price of other number of cans. She said that they need to continue the

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table, but when the researcher asked whether they could determine from the existing table which one was cheaper, they answered yes. Thus, in the end of the discussion, the researcher remarked that there was no need to continue the table if they already knew which the cheaper one was.

It is not clear why Livia wanted to continue the table. It may be that she thinks that the result can be different for a different number of items, or it may be that she just wants to end the table with the same number of sodas (line 16, 17).

From this discussion, we can see that the table helps the student to determine the cheaper price of two different items. Based on the pretest and interview in the beginning of this study, the students already had some idea of comparing something. They could compare which one cheaper if they had the same price for different number of the items or had the same number of items for different price. The students' difficulty in the pretest was they didn't know how to make the price or the number of items the same. The ratio table can help them to do this.

CONCLUSION

The result of this study shows that introducing the ratio table within a money context may support students to develop their proportional understanding. The ratio table helps the students to organize the information from the problem and lets them develop calculation strategies like adding up, or doubling. In line with that, the money context keeps the students thinking in a proportional way. However, the problems should be expanded to other contexts to prevent students to limit the application of proportionality.

Limitation

Because of the study was conducted in a class which was only consist of 29 students Grade 4, we cannot generalize the result or the conclusion of this study in the world wide. However, we still can use the design to find out whether it works or not if it implemented in other place with different students.

Suggestion

For the next research, this design may be implemented for the younger children, such as the second or the third grade because the ratio table also can be taught as a simple tool to do calculations. The lower graders are familiar with the money context, so they may not have any difficulties to work on it as long as the numbers are appropriate.

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