LEARNING ONE-DIGIT DECIMAL NUMBERS BY MEASUREMENT ACTIVITY

Puji Astuti
Surabaya State University
P.Astutipuji@gmail.com

Abstract

It is a prerequisite for students to have an understanding of the concept of fractions to learn decimals. Also, modelling the decimals, like using the number line, supports students to see where the decimal numbers are among the whole numbers and the decimal numbers itself. Therefore, this study aims to support children’s understanding of the meaning of decimals by seeing: 1) the tenths and hundredths of decimal numbers by the context of measurement; 2) the representation of decimal numbers on number line. With this aim, design research method is used in this study. We develop an intervention to improve conventional teaching approaches in decimals through instructional activities on measurement activity and game context. By the measuring activity, students are hoped to be able to relate tenths and hundredths to one-digit and two-digit decimals. Besides, the number line through game activity is hoped to contribute to students’ development of decimal concepts. The instructional activities are designed and compacted in the Hypothetical Learning Trajectory (HLT). The HLT was carried out in two cycles involving the 4th grade of elementary school students. Data of the learning activities collected from videotaping along teaching process, field notes by the researcher, and the completed learning worksheet are used to refine the HLT. This article describes and discusses the learning activity of 2 meetings of cycle 2 of our focus group students of grade 4 elementary school in Surabaya: SDIT Al Ghilmani. The data indicated that the learning activities promoted the students’ understanding of one-digit decimal numbers.

Keyword: measurement, decimal numbers, number line

INTRODUCTION

Background

The concept of decimals is included in the mathematics curricula and is considered to be of great significance especially due to its application and use in everyday life (Michaelidou, Gagatsis, & Pantazi, 2004). In Indonesia, the concept of decimal numbers is stated in the curriculum, Kurikulum Tingkat Satuan Pendidikan (KTSP), for the fifth grade of elementary school. It is important for students to learn decimals also because the decimal concepts support learning other mathematics topics especially measurement of length and area, and arithmetic. In brief, decimals are essential in both mathematics and its applications.

Nevertheless, some recent studies reported students’ weakness on decimals (Bell, Swan, & Taylor, 1981; Moloney & Stacey, 1997; Stacey et al, 2001; Steinle, 2004; Lai & Tsang, 2009; Sengul & Guldbagci, 2012). Most common areas of problems in learning decimals from the literatures are weak understanding of place value and weak...
notions of the magnitude of decimal numbers. Grossman (in Steinle, 2004) indicated students’ weakness on magnitude decimal numbers with data that only 30% of a large sample of students was able to choose the smallest decimal number from a list of five different decimal numbers. While weak in place value, for example, is students assume that the relationships within the whole number system can be transferred unaltered to the decimal number system. For example in whole numbers, 20 is ten times greater than 2, but it is not correct to say that 0.20 is ten times greater than 0.2 (Moskal & Magone, 2001). As is often the case, when being asked to compare 2.25 and 2.3, students would choose 2.25 as the larger number because 25 is bigger than 3.

These facts lead to investigation what and how decimals are usually taught, and arises a need for intervention for teaching and learning building an understanding of decimals. Lai and Tsang (2009) outlined that in learning decimals, students in primary school are pushed to learn the four operations of decimal numbers rather than to understand the meaning of decimal numbers. According to Lai and Tsang, focussing on students’ computational skills on four operations of decimal numbers is merely rote learning and devoid of any meaning. In a similar vein, I argue that it is dangerous to learn operations on decimals without understanding the meaning of decimals because students might expert only in calculation but do not have convincing ideas what they are doing. Unfortunately, teaching emphasizing computational skills in learning decimals also happens in Indonesia. My investigation starts with an idea that the textbooks used for teaching-learning might reflect what teacher teaches in mathematics classroom. The observation about students’ textbooks (see Sumanto, Kusumawati, & Aksin, 2008; Soenarjo, 2008) gives impression that students are taught about the concepts of decimals in a very formal way. A procedural way was introduced to find decimals: changing a fraction by multiplying the numerator and denominator with the same number, or using a traditional method division.

This kind of learning decimals could be reasons behind students’ misconceptions on decimal numbers. While it is known many studies have investigated that students still have to struggle on understanding decimals, it is not known how teachers can support students to learn decimals. Hence, we aim to develop teaching sequences to avoid the misconceptions, not necessarily teaching which opposes the misconceptions one by one but teaching which supports building an understanding the meaning of decimals.

At present, international studies report teaching practices designed to avoid confronting difficulties on decimal numbers (Helme & Stacey, 2000; Huang, Lium, & Shiu, 2008; Yildiz, Baki, Aydin, & Kogce, 2010; Lachance & Confrey, 2002; Brousseau, G., Brousseau, N., Warfield, 2007). However, research in Indonesia about intervention in teaching learning decimals has been minimal. Pramudiani (2011) implemented activities about weight and volume measurement to promote the students’ notion of decimals in which the students could discover decimals and determine their position in between two consecutive whole numbers (on the scale). Although it is shown that the activities can help students to understand the magnitude of decimal numbers, the understanding of the relation between fractions and decimal numbers has not been explored further. Hiebert (in Lai & Tsang, 2009) argued that a student who fully understands the concept of decimals would be able to provide meaning for the
symbol notation and provide the reasons for the symbol rules such as dividing the numerator by the denominator to write a common fraction as a decimal. Therefore, the present study differs in ways of the focus; we think to support children’s learning decimals with aim students’ understanding of: 1) the meaning of decimal notation; 2) fractions and decimals are similar thing.

**Research Questions**

The general research question for this study is: “How can measurement activities facilitate the students’ notion on decimal numbers?” In answering this research question, the following sub-question are to be investigated:

1. How can students develop an understanding of one-digit decimals?

   The answer of the question would be description on how making partition into ten of measurement unit activity leads students to an understanding of one-digit decimals. This is done through the thinking process of finding length of things to giving them decimal notation.

2. How can students develop an understanding of two-digit decimals?

   We investigate on how making partition into one hundred of measurement unit leads to students’ understanding of two-digit decimals.

**Aim of the Research**

The present study concerns on doing intervention by learning experience for students to construct the knowledge about decimals. Students are hoped to be able to relate tenths and hundredths to one-digit and two-digit decimals. However, the study does not discuss about conversion of fractions to decimals. We propose an activity to measure length, in which students need to cooperate in investigating the best way to represent a part of a whole number. The reason an activity to measure length is chosen to learn about decimals, is because a measurement scale could be easier to relate to the number line. The present study also proposes number line to support students’ understanding on decimals. As Thomson and Walker (in Michaelidou, 2008) suggested that the number line contributes to the development of concepts not only related to the identification and comparison of decimals but also to the ability to perform operations. A recent study by Michaelidou et al (2008) also showed that students succeed in representing the concept of decimals using a number line. This paper will present the learning process of the students to answer question research 1.

**Research Method**

This study aims to develop intervention to improve conventional teaching approaches in decimals through measurement context. Therefore, “design research” method is used in this study. The design research in this study is an iterative process which refers to the research incorporates a cyclic approach of preparing for the experiment, teaching experiment, and retrospective analysis (van den Akker, Gravemeijer, McKenney, & Nieveen, 2006).

In the preparation phase, some data collection will also be conducted, such as classroom observation, teacher interview, and pre-test. The HLT then was implemented in two cycles: preliminary teaching experiment (cycle 1) and teaching experiment (cycle 2). During the implementation of the HLT, data was collected by
video recording of the whole class learning process, the video recording of the focus group, and the field notes.

This paper will explain about the teaching and learning process of cycle 2 for meeting 1 and meeting 2 which is about one-digit decimals.

MAIN SECTION

Here we present a description of learning decimal numbers of students in the 4th grade of SDIT Al-Ghilmani and more focus to our focus group students: Aam, Daya, Rifan, and Shofil.

Pre-Test Decimal Numbers

The pre-test questions were designed to know if the students have preliminary knowledge to learn about decimal numbers. The problems included were fair sharing problems, guessing number between two whole numbers like is there any number between 3 and 4?, numbers on number line, and measurement.

In fair sharing problems, the students divided the bar properly and wrote correct fraction notation. However, some still used words to notate the fraction, for example 'seperempat (a quarter), setengah (a half), and sepersepuluh (a tenth)'.

Many of the students could not guess decimals on guessing number problem. They answered 'no numbers' or with correct fractions, like 'three and a half', only one student, Raju, answered with a decimal number '3,5'.

All students did well writing consecutive whole numbers on number line. But most did not recognize decimal numbers labeled by a letter put in the middle of two numbers on number line. Only five students: Daya, Arya, Shofil, Daffa, Alawi correctly identified decimals on the number line, such as 1,5; 2,5; 3,5 and 4,5. The others just said \( \frac{1}{2} \) or 'a half' or incorrect fractions, like \( 1 \frac{1}{5} \).

When the students did measurement problem, they also recognized decimals. Yet, their decimal numbers were not correct (see figure 1). They probably emerged the decimal numbers because their experience and knowledge using a ruler. The one and only who made 10 partitions on the paper strip drawn and notated with correct decimals was Aam.

![Figure 1: Daud’s misconception on decimals](image)

From all the data above, the students seemed to have good understanding on fractions. The students' responses of the tasks also displayed how far they knew about decimals. In abstract problem, like guess what numbers between 3 and 4, it was found that the students did not yet recognize decimal numbers. However, it is
known that with number line model there were some who had recognized that a half was .5. In addition, from our measurement problem, we could see the students would use decimal numbers when the result does not fit.

Before this test, the focus group students were already chosen: Aam, Daya, Rifan, and Shofil. It was found the four students familiar with number line. Aam was accurate than the others on dividing the bar, he used ruler, and wrote fractions properly. He also showed his recognition of decimals although in some answers were incorrect. Daya knew fractions and used decimals but did not recognize any numbers between 3 and 4. Rifan was good at fair sharing problems, but could not appear decimal number. Shofil was also good at fractions, appeared decimals in his answer some correct and some not.

Overall, it was concluded that the students had enough prior knowledge to follow our learning series. They had knowledge about fractions: dividing a bar into equal parts and gave name the parts with fractions, and about number line.

**Meeting 1: Measure things until they decide to use 10th**
The first meeting aims to encourage students to use tenths in notating length of things before they turn into a decimal quotient later. There are two main activities: first, they measure table with their span and investigate why the measuring would give different length result for same objects, so how we could get the same measuring result? The discussion would end by introducing the paper strip as the standard length of their span to do measuring. Second, they explore measuring with the paper strip and discuss what would you do if the paper strip does not fit for many times? This leads them to symbolize the length as unit-fractions.

In the first activity, not as expected in the HLT students would get various length when measuring a table with a span, in the class, the students measured various things with various non-standard measurements: span, thumb, and foot. Although so, they who measured a table compared, in the whole class, the different result of their measurement: 3 spans, 1 step, 34 thumbs, 3 feet, 50 little fingers, and 3 caps. Being asked why different, the focus group students argued it was because of different ways of measuring and different children who measured.

The focus group students suggested using a ruler and a paper strip so that the measurement result would be the same (these students knew the paper strip because they opened the next page of their worksheet). Although they were cheating, it was known that they gained idea for a refinement of a standard measurement tool. Other groups discussed why different, because: the measurement tool they used were different; there was a long span and a short span. One group suggested using a measurement tool which has the same length.

In the next activity, as the students suggested using the same measurement tool, expected in the HLT, hence the paper strip was introduced, representing a span. They in group measured things in the classroom. They were struggling to say the rest length; the length which was not exact whole paper strip. In the focus group, Daya initially measured the length of a table and found that the table did not fit completely the paper strip. He and friends discussed how to say it and started to use unit-fractions (see fragment 1).

<table>
<thead>
<tr>
<th></th>
<th>Aam</th>
<th>It is a quarter, it is a quarter <em>(told the rest measuring table)</em></th>
</tr>
</thead>
</table>
In fragment 10, the students showed their mastery of naming fraction parts but confused because the length was mixed fraction (fragment 1 line 13-17). The following discussion showed they used both fraction and decimal numbers to say the length. (See fragment 2).

The teacher then conducted classroom discussion and invited the students to compare their approach dividing the paper strip. This actually was way to compare different fractions of the length of same object. The different fractions would puzzle the students whether the length is actually the same, so they need a refinement of fractions they need to use. However, only Nabil's group, with 8 divisions, came in front of the class with fractions. The classmates seemed confused how to notate the rest: only in terms of halves, quarters, making incorrect fractions and even pretended that the length was exact the paper strip. Thus, the teacher directly asked the students to divide the paper strip into 10 parts. He explained this decision was made because of our ten-base system.

The teacher drew on the whiteboard the paper strip divided in 10 parts and discussed the fraction of each part. The students practiced to say the tenths. Then the
teacher measured the height of the whiteboard and guided the students to name the length in tenths. The plan to ask the students to measure again with divided paper strip was cancelled due to time limit.

From the findings so far, it was observed that the focus group students were in their phase developing the fraction language to represent the parts of the paper strip. Another fact, they performed decimals in incorrect manner. Fragment 10 and fragment 11 give information how they misunderstood naming with fractions and decimals: 2 paper strips and \( \frac{1}{5} \) was said as \( \frac{2}{5} \) and was written 2,5. It was also noticed the other students had problems naming fractions. Overall, the students still had problem: 1) name the fractional pieces, especially writing mixed fraction and 2) make incorrect equivalent relation of fraction and decimal number.

In conclusion, however, this lesson 1, with the paper strip, gave experience for the students measured things, divided the paper strip, gave estimation and discussed its rest with fraction language. Although it was the teacher telling them to divide the strip in 10, from the process gaining fraction it is known that they developed the sense of tenths. The instructional activities were set for students’ understanding of decimal notation by its relation with tenths. Therefore, although the decimal numbers were already mentioned by students at initial phase, it was decided, as in HLT, to begin with students’ good sense of tenths.

**Meeting 2: One-digit Decimal Numbers**

In the second lesson the context of measurement was again presented to generate decimal notation. They were asked to measure things in the classroom with divided-10 paper strip and notate the result in terms of fractions. Then, the question asked to them was to leave out the fraction tenth, what would you write?

At the beginning of the class, the students divided the paper strip into 10 parts by themselves. Some folded it and some used ruler making each part 2 cm (the paper strip was 20cm). It was noticed that students of our focus group used long names for the fractions that came up, but improved their fraction understanding, like the length of a stake was “8 more \( \frac{2}{10} \) paper strips”. They explained, in worksheet, their numbers as follow, 8 was 8 paper strips and \( \frac{2}{10} \) was 2 parts which was folded by 10 (they meant 2 pieces of 10). While, there were some students excluding the focus group struggled making the fractions in term of the length of a paper strip, for instance the length of a cap was written as 1 and 5 paper strip (the students intended to say 1\( \frac{5}{10} \)), and the height of a table was written \( \frac{20}{20} = 2 \) (the students counted the small part of the paper strip, so 2 paper strips was 20 small parts).

The problem ‘what do you write for the length if we do not use tenths?’ was supposed to be group discussion, but became whole class discussion. The teacher asked ‘if we erase tenth in 8 more \( \frac{2}{10} \) paper strips’. The solutions proposed by some children written on the boards were discussed in whole class. They answered with informal and long names: 8 and more 12 paper strips, 8 and more 2 parts from 10 parts of the paper strip, 8 and more \( \frac{1}{5} \) paper strip (Aam tried to make different answer by using equivalent fraction), and 8,2 paper strip. The teacher took their responses into a discussion leading to decimals (see fragment 3).
Soon, our students in the focus group were able to communicate their measuring result, using fraction tenth before, in decimal numbers (see figure 2).

Figure 2: Focus group students transformed tenths into decimal numbers

While discussing tenths to decimal numbers, the confusion of focus students appeared how to name the length with decimal number for small things, like the length of an eraser, $\frac{6}{10}$ (see the fragment 4).

<table>
<thead>
<tr>
<th>1</th>
<th>Researcher</th>
<th>If $\frac{6}{10}$, what is the decimal form of it?</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Rifan</td>
<td>Six comma ten $[6,10]$</td>
</tr>
<tr>
<td>3</td>
<td>Researcher</td>
<td>Shofil, $\frac{6}{10}$ is already 1 or not?</td>
</tr>
<tr>
<td>3</td>
<td>Shofil</td>
<td>Not yet.</td>
</tr>
<tr>
<td>4</td>
<td>Researcher</td>
<td>So, how do you notate it?</td>
</tr>
</tbody>
</table>
All above, it is noticed the focus students, and the rest, struggled to transform from informal to formal notation and from tenths to decimal numbers. The conjecture in HLT students would use strip, apostrophe, as separator wholes and tenths did not appear. How the students could come with the idea using comma symbol quickly? They were already familiar with decimal numbers. As appeared in the pre-test and meeting 1, if the length could not be said in ones, the students tended to say in decimals, though in incorrect manner. In this second lesson, their prior knowledge of decimals was intervened by the idea that the decimal is another way to write fraction tenth.

The fragment 12 shows clearly now the students could reason what they meant by 8.2 and why comma was used. Surprisingly, they saw the number relationships, which the ones and the tenths. Moreover, the confusion notating length of small things with decimal numbers (shown in fragment 4) actually shows how the students were gradually building idea of the meaning of decimal numbers.

CONCLUSION

The conclusion for meeting 1 is that the context supports the students to investigate how to notate the rest of things measured with the paper strip. The students initially suggested fractions and decimals though in incorrect manner to notate the rest. Later, they argued the idea to use the same fraction. Then ten-partition was introduced directly by the teacher not by the students' side. For the next cycle, it should be prevented that the teacher tells much about the idea. Let the students discuss first what to do if they use different fractions. Also, some practical things are needed to notice:

1. Let the students measure a table with their span, rather than students measure many things and at final only compare one thing, it is better for them to be instructed from the beginning to measure one thing only. It is also to make them focus in measuring one thing and would discuss why the result of their measurement are different

2. The teacher should notice that there would be students who did not care about the rest. Students just say the length in terms of the paper strip, for example 3 paper strips, without counting on the rest.

3. Some students still use ruler to measure things.

The conclusion for meeting 2, the context promotes the students to use correct names of the fractions in notating measuring result and supports them to make correct interpretations of decimal numbers. The students initially proposed long names when asked not to use fraction, later they suggested using comma to separate the
length along the paper strip and the parts. Although so, it was noticed that the students needed considerable time to come to the idea of using decimal numbers. But at least, it was observed the students tried, argued, and discussed why they need to use comma.

The general conclusion to answer sub research question ‘How can students develop an understanding of one-digit decimals?’ is that the students gradually developed their understanding of decimal one-digit. The context brings the students from misunderstanding of decimal numbers (shown in pre-test), puzzling naming with incorrect fractions, here in the situation of measuring result, to using tenths, then to using decimal numbers. This learning should be supported by the discussion among the students themselves. Let the students argue and give their opinion; also, the teacher should guide them by giving good question like, why you use this? What does it mean?

REFERENCES


